Phenomenology of the size effect in hardness tests with a blunt pyramidal indenter

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Further detailed analysis of the indentation size effect exhibited by some single-phase metals leads to a new, very accurate, descriptive equation. This affords consistent and realistically low evaluation of macrohardness from micro-indentation test data.

The indentation size effect exhibited by fused silica is also matched precisely by the new description, demonstrating a common phenomenology regardless of the different micro-mechanisms sustaining indentation.

Comparison of data from standard and low-load Vicker's tests with data from ultra-micro-indentation with a Berkovich indenter establishes continuity of a monotonic size effect throughout the entire range of indent size.

The observed size effects are consistent with the projected refinement of a previously proposed model of indentation that attributed the effect to varying importance of the constrained flexing at the perimeter of the indent.

The magnitude of the size effect appears to be a measure of the resistance to strain concentration in the perimeter flexure zone. The large size effect for eminently plastic metals indicates that restricted micro-deformation capability is not the major cause.

Nomenclature

a	numerical constant (1.854 for Vicker's hardness)
d	observed diagonal length of square Vicker's indent (μm) (see L)
F	force (mN)
H	apparent hardness (defined, conventionally, as load/area; kg mm^{-2})
H_∞	minimum macrohardness
H_0	estimate of H_∞
H_V	Vicker's hardness ($= 0.927 \times$ mean pressure expressed as kg mm^{-2})
H_{V20}	Vicker's hardness value for test with 20 kg load
L	indentation test load (kg) (suffixes identify associated data)
m	the "Meyer index" (varies with scale of indentation)
P	penetration of indenter (nm)
β	size effect parameter for Vicker's indentation
δ	calibration offset in value of P
ε	strain
η	notional error in value of d
λ	size effect parameter for depth-sensing indentation
σ	stress

1. Introduction

Although indentations made with a quasi-rigid pyramidal indenter appear to be geometrically similar [1–3], a size effect in small scale indentation hardness testing has long been recognized [4–7]. Examples of the effect in low-load (10 g–1 kg) through to standard Vicker's hardness tests of commercially pure aluminium are shown in Fig. 1. Even greater disparity between apparent hardness in micro-indentation tests and the corresponding macro-indentation hardness is a serious problem. Naturally, many attempts have been made to explain the size effect, and many possible factors have been identified, but without satisfactory resolution: variation of apparent hardness with indent size is still reported as the end result of testing.

Several descriptions of the size effect have been proposed; but they have proved to be no more than approximate. Meyer analysis [4] according to

$$H_0 = \frac{aL}{d^m} \qquad (1)$$

has often been applied to pyramidal indentations, but has been shown to be inaccurate even for the size range of standard Vicker's tests [8]. Other common equations, embodying a measurement 'error' parameter or a quadratic relationship between indent dimension and indenting force, have been shown to be precisely equivalent [8, 9]. Therefore it is sufficient, and appropriate for a size effect, to consider only the

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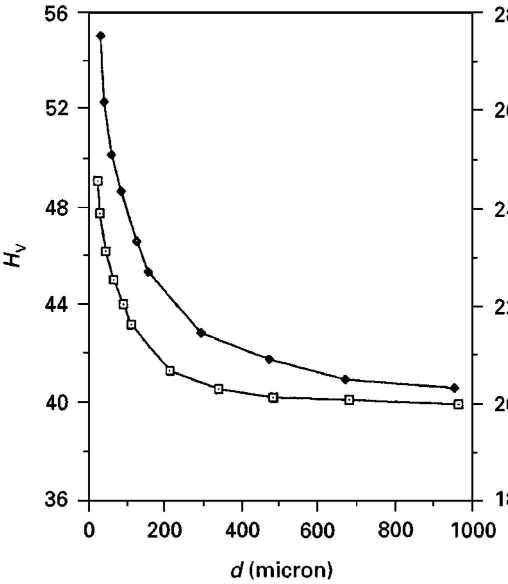

Figure 1 Indentation size effect for Vicker's hardness tests of aluminium. (□) hard; (◆) soft.

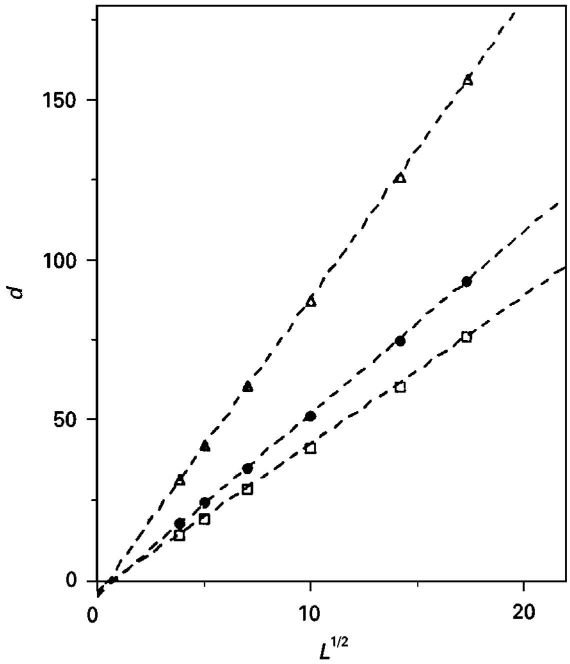

Figure 2 Evaluation of η in Equation 3. (△) Al; (●) Cu; (□) Fe.

description proposed by Tate [5].

$$H_0 = \frac{aL}{(d+\eta)^2} \tag{2}$$

This equation may be rearranged into a form suitable for linear regression analysis and thus for the evaluation of η

$$d = \left(\frac{a}{H_0}\right)^{1/2} L^{1/2} - \eta \tag{3}$$

For Vicker's hardness tests with load down to 15 g this relationship might appear to be quite accurate. Examples are shown in Fig. 2. However, the apparent linearity has been shown to be deceptive; even at the relatively coarse scale of "low-load" testing, refined analysis according to this model failed to account for the persistent elevation of calculated macro-hardness for smaller indentation [9]. In ultra-micro-indentation testing the equivalent relationship between P and $F^{1/2}$ is often clearly non-linear, and amenable to different interpretations [10].

Variation of apparent hardness with indent size was long thought to be caused by experimental error, or to elastic recovery of the indent, but gradually a true size effect came to be recognised and explanation has been sought in variation of micro-mechanical processes sustaining deformation in small volume [6, 7, 11]. A major weakness of this approach is that it seems very unlikely to explain the continuous gradation of the size effect well into the range of standard Vicker's hardness testing of metals (See Fig. 1), for which the deformed zone may extend a millimetre or more (some million atom distances).

Recently it has been confirmed in detail that the magnitude of the size effect is governed by the strain hardening propensity of the material indented and by friction [10, 12–15]. In the light of this knowledge it has been postulated [16] that the size effect is a manifestation of flexing at the perimeter of the indent, where the indented surface is aligned with the face of the indenter. In an 'ideal' plastic material deforming without strain hardening, the perimeter "hinge" zone might be vanishingly small – as indicated by shear line field constructions [17] – but in real materials, especially strongly strain hardening metals, such strain concentration is unlikely.

For the indentation boundary conditions to cause a size effect, the perimeter zone dominated by flexing must vary in relation to the overall deformation pattern in a rather special way. If the *proportions* are constant, a size effect is improbable; but a constant *magnitude* of this zone would exceed the size of very small indentations. Therefore the relative size of the zone must change in some intermediate way. The problem was recognized when the effective size of the perimeter "plastic hinge" zone was first calculated [16], and it has since been established that the value of an error parameter such as η does change with indent size [9]. Thus, the size effect was demonstrated to be even more complex than earlier supposed.

This finding has a significant bearing on micro-indentation and ultra-micro-indentation hardness tests. Such tests are subject to pronounced size effect, but the observed variation is severely truncated. Consequently the estimation of a characteristic minimum macro-hardness is uncertain unless the established relationship is very accurate and continuous throughout the full indentation size range.

The "plastic hinge" model of indentation [16] provided a useful explanation of the size effect for 'low-load' indentation of metals. However, extrapolation to ultra-micro-indentation requires more precise knowledge of the phenomenon. An opportunity to explore the size effect in some detail was afforded by availability of test data representing a very wide range of indent size: Vicker's hardness tests with a load range of 15 g to 20 kg [12] and comparable ultra-micro-indentation tests with force down to 2 mN [18]. As

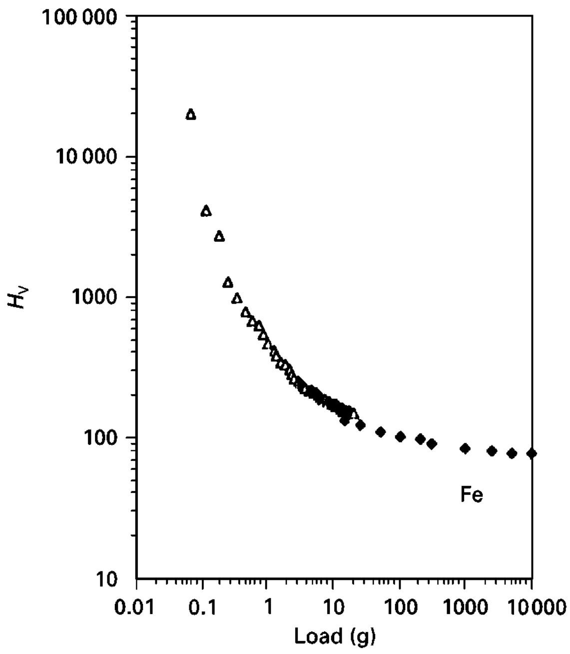

Figure 3 Continuity of indentation size effect for iron over a wide range of test conditions. (△) Berkovich; (◆) Vicker's.

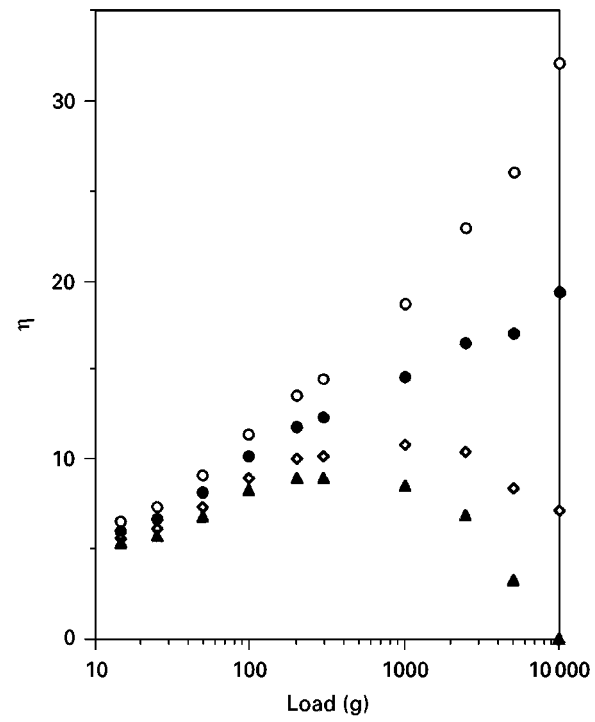

Figure 4 Effect of varying estimate of macrohardness on evaluation of η according to Equation 4. H_0: (○) 19; (●) 19.5; (◇) 20; (▲) 20.3. Data for soft aluminium.

shown in Fig. 3, only a slight offset is detectable in the otherwise continuous relationship between apparent hardness and indentation force for specimens of iron.

Ultra-micro-indentation data for fused silica are also analysed to compare the similar size effects exhibited by obviously dissimilar materials. Earlier findings [10] from analysis of reported ultra-micro-indentation data are re-examined.

2. Analysis

2.1. Low-load Vicker's tests

When the characteristic minimum macro-hardness H_∞ is known, the complexity of the indentation size effect may be revealed by calculating the expected variation of η in Equation 2 according to

$$\eta = d\left[\left(\frac{H}{H_\infty}\right)^{1/2} - 1\right] \tag{4}$$

In recent analyses [9, 14] it was assumed that the hardness value determined from tests at 20 kg load would be a good approximation to H_∞, but the effect of error in this approximation was not pursued. In fact, exploration of this effect provides a revealing insight into the indentation size effect.

Although the original finding, for several metals indented with a Vicker's indenter under loads in the range 15 g to 20 kg, was that calculated values of η increased and then decreased with increasing test load, it is obvious from Equation 4 that the decline in values of η as $H \to H_\infty$ is hypersensitive to the value assumed for H_∞ – and the observations of the size effect indicate that H_{V20} is very probably an overestimate. Adopting a slightly lower estimate of H_∞ transforms the variation of η into a monotonic relationship with the indent size, d. This is demonstrated in Fig. 4, using data for soft aluminium as an example. These data were chosen because the relatively large indents minimize the possible impact of experimental error.

Uncertainty in the estimation of H_∞ was overcome in an earlier analysis [9] by calculating "instantaneous" values for η according to

$$\eta = \left[\left(\frac{L_1}{L_2}\right)^{1/2} d_2 - d_1\right] \Big/ \left[1 - \left(\frac{L_1}{L_2}\right)^{1/2}\right] \tag{5}$$

The calculations for several metals yielded persistent trends to continuously increasing η with increasing d. Averaged results for similar specimens, classified according to strain hardening propensity, showed clearer relationships. Re-examination of these data reveals that the relationship between η and d is of the form

$$\eta \propto d^{1/3} \tag{6}$$

The averaged data are re-presented in Fig. 5 in log versus log format, together with the appropriate linear regression equations to verify this observation.

Modification of Equation 2 to accommodate this new information leads to

$$H_0 = \frac{aL}{(d + \beta d^{1/3})^2} \tag{7}$$

The parameter β may be evaluated by regression analysis of data for several test loads, utilizing a derived linear form of Equation 7

$$d^{2/3} = \left(\frac{aL}{H_0 d^{2/3}}\right)^{1/2} - \beta \tag{8}$$

Examples of these relationships, for the three annealed metals among the specimens referred to above, are shown in Fig. 6.

Calculation of H_0 according to Equation 7 yields close to constant values, even down to the smallest

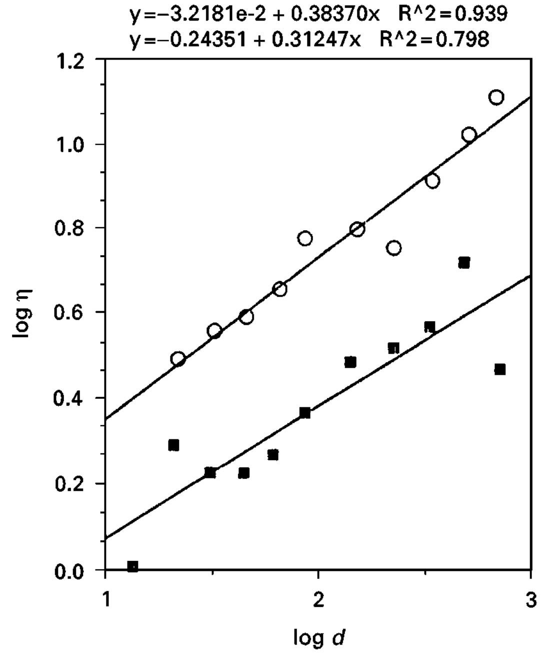

Figure 5 Relationships between average values of η and *d* for three metals in (○) soft and (■) hard conditions.

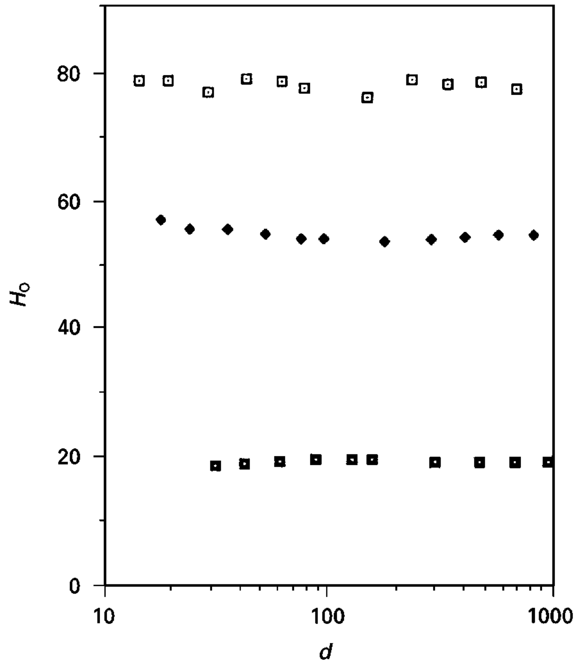

Figure 7 Calculated macrohardness for three soft metals according to Equation 7. (□) Fe; (◆) Cu; (■) Al.

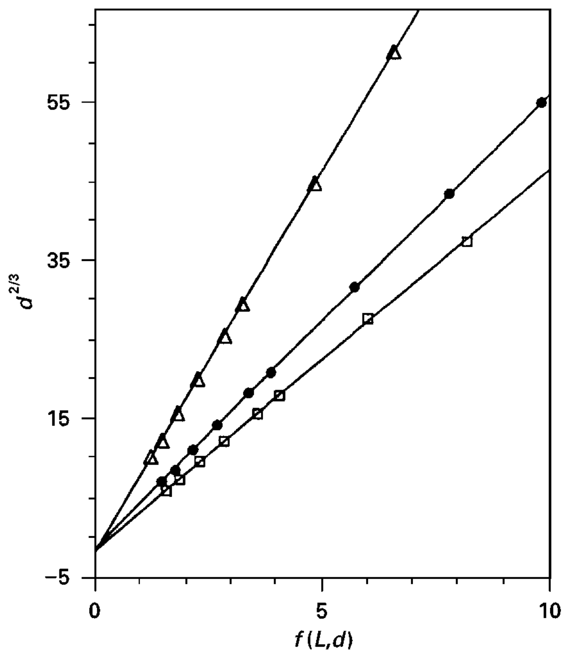

Figure 6 Linear relationships according to Equation 8 for three soft metals, showing evaluation of β. (△) Al; (●) Cu; (□) Fe.

indentation. Fig. 7 shows the results for these three metals. Evidently Equation 7 is an accurate description of the indentation size effect in Vicker's hardness tests of these metals tested with loads down to 15 g. The mean estimated value of each minimum macrohardness is slightly lower than the H_{V20} value; and divergence from the mean, previously reckoned at up to 25%, is only 2 or 3% and randomly distributed. This appears to reflect the true low level of experimental error, now properly differentiated from the larger variation with size of indent.

2.2. Ultra-micro-indentation of iron

Very fine indentation hardness tests are made with an instrument that measures penetration of the indenter rather than the size of the indent [19–23]. The Berkovich triangular based prism indenter is preferred to the Vicker's square form at this fine scale, because it is difficult to maintain accurate shape of the latter at its tip. The process is attended by two difficulties peculiar to the method: establishing the datum of first contact and accounting for imperfection of the indenter tip.

In principle, it is possible to calibrate the measurement of indenter position to obtain accurate penetration data (and this is, of course, done); but reported test results display a considerable size effect that is reducible by recalibration [10], and quoted hardness values are often much higher than expected macrohardness. There appear to be grounds for reasonable doubt about the appropriateness of calibration procedures for ultra-micro-indentation tests. In these circumstances, some additional complication in analysing the size effect may be expected.

Ultra-micro-indentation test data [18] for the interior of a ferrite "grain" in a very-low carbon steel were examined for comparison with the low-load Vicker's data for pure iron, described above. The data were found not to conform with Equation 3, which is the usual calibration format. Fig. 8 shows that the size effect cannot be described simply by a linear relationship of that form. (This is important, but not new: the manual for the testing instrument describes a nonlinear relationship.)

The ultra-micro-indentation data were next analysed according to Equation 8. Unlike the low-load data for iron discussed above and shown in Fig. 6, the relationship was not linear – especially for the finest indents. Considering the recognized calibration

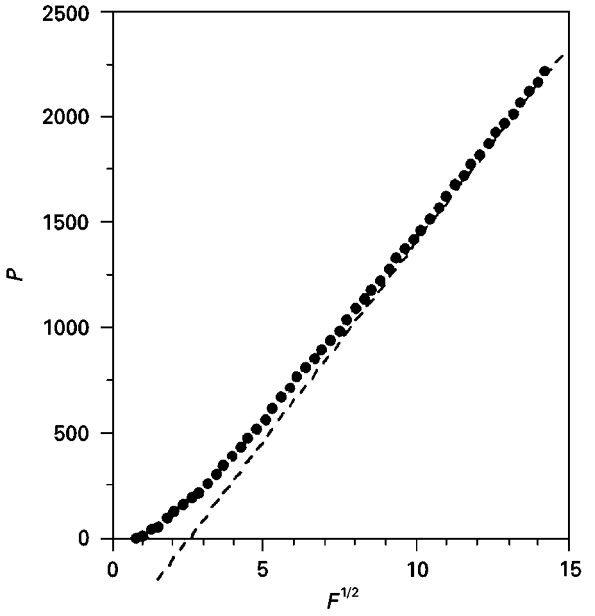

Figure 8 Deviation of data for ultra-micro-indentation of iron from relationship according to Equation 3.

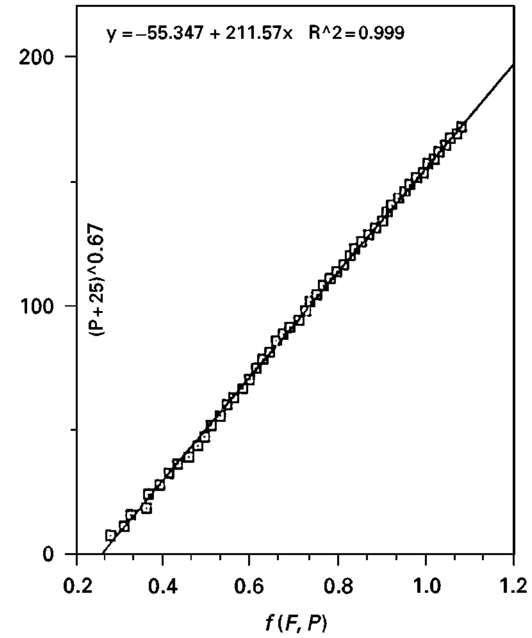

Figure 9 Linear relationship according to Equation 10 for ultra-micro-indentation of iron.

difficulties for depth-sensing instruments and the non-linear relationship corresponding to Equation 3, it seemed that a possible cause of this non-linearity was an offset error in the calibration of penetration.

Incorporating an unknown supposed penetration error into Equation 7, and modifying for the conventions of ultra-micro-indentation testing (in which force and penetration are measured), gives

$$H_0 = \frac{aF}{49[P + \delta + \lambda(P + \delta)^{1/3}]^2} \qquad (9)$$

and Equation 8 becomes

$$(P + \delta)^{2/3} = \left(\frac{a}{49H_0}\right)^{1/2} \left(\frac{F}{(P + \delta)^{2/3}}\right)^{1/2} - \lambda \quad (10)$$

For a data set embracing several values of F and P, Equation 10 is readily solved numerically by iterative regression incrementing a given value of δ until the correlation coefficient reaches its maximum value.

In the example of the ultra-micro-indentation of a ferrite 'grain' interior, the calibration 'error', δ, was found by this method to be ~ 25 nm. This small adjustment was sufficient to produce a linear relationship of the form of Equation 10 with $r = 0.999$. This is shown in Fig. 9. The value of λ was found to be $\sim 55\ \text{nm}^{2/3}$ and, substituting these values in Equation 9, the estimate of characteristic macro-hardness, H_0, was virtually constant at $\sim 87\ \text{H}_\text{V}$ and closely comparable with the lower limit of standard Vicker's hardness for iron. The constancy of this estimate and the greatly varying high level of apparent hardness are compared in Fig. 10. (cf. Figs 3 and 7) Evidently Equation 9 is a very accurate description of the indentation size effect for this ultra-micro-indentation of iron with a Berkovich indenter.

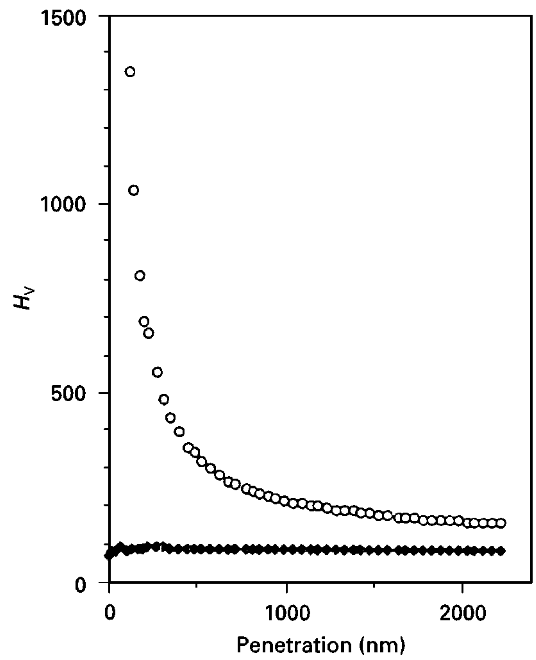

Figure 10 Apparent hardness and calculated macrohardness for ultra-micro-indentation of iron. (○) H; (◆) H_0.

2.3. Ultra-micro-indentation of silica

The steps explained above for analysing the ultra-micro-indentation of iron were repeated with data [24] for fused silica. Again, it was found that the data did not conform linearly with Equation 3, though the deviation was less than for iron. See Figs 8 and 11. Better agreement was obtained with the relationship of Equation 10. In this case the calibration "error", δ, was found to be ~ 19 nm, which is similar to that for iron, suggesting comparable experimental error although a different machine was employed. The value of λ, however, was found to be much lower at $\sim 6.5\ \text{nm}^{2/3}$. Substituting these values into Equation 9,

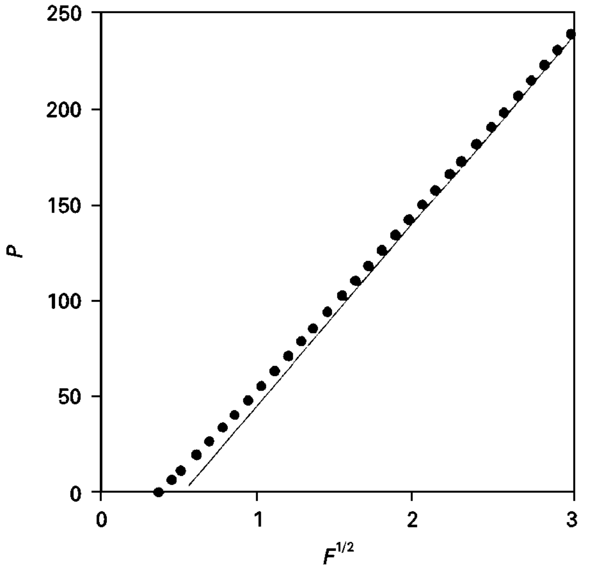

Figure 11 Non-linearity of relationship according to Equation 3 for fused silica.

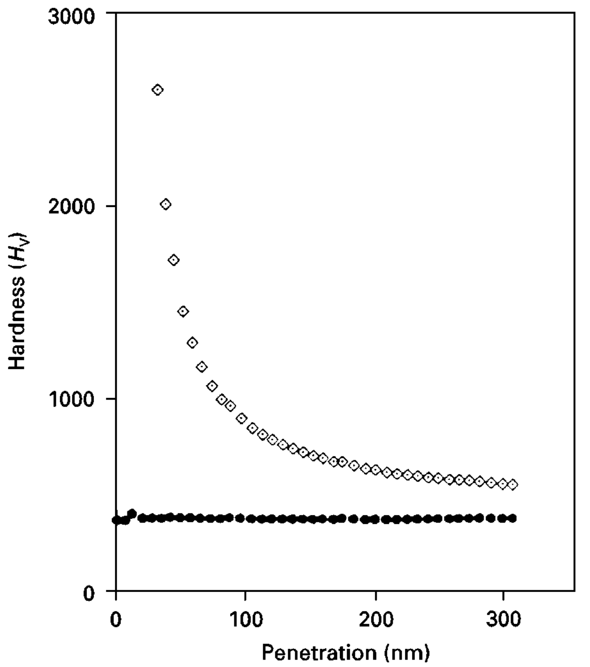

Figure 12 Apparent hardness and calculated macrohardness for fused silica. (♦) H; (●) H_0.

the estimate of characteristic macro-hardness, H_0, was virtually constant at $\sim 360\ H_V$.

Evidently Equation 9 is also a very accurate description of the size effect for ultra-micro-indentation of fused silica with a Berkovich indenter. The constancy of the estimated macro-hardness and the greatly varying high level of apparent hardness are compared in Fig. 12. The similarity with Fig. 10 is striking, but the scale of indentation is considerably different – and the much weaker size effect is represented by an order of magnitude difference in the values of λ.

3. Discussion

3.1. Analysis

Earlier calculation according to Equation 5 of "instantaneous" values of the indent size error, η, afforded an insight into the complexity of the size effect but, as demonstrated [9], could not obviate the gross residual variation of H_0 even for low-load Vicker's indentation. However, since it is now clear that Equation 2 (from which Equation 5 is derived) is not an accurate description of the variation of apparent hardness for small indents, it must be accepted that those estimates of minimum macro-hardness, being hypersensitive to error in the value of η when d is small, were invalid.

A recent demonstration [10] that the size effect in ultra-micro-indentation can be largely explained by assuming a relationship of the form of Equation 2

$$H_0 = \frac{aF}{49(P+\delta)^2} \tag{11}$$

is also seen to be suspect since Equation 2 has been discredited. In fact, data for the indentation in aluminium clearly did not fit Equation 11. It is now apparent that descriptions such as Equations 2 and 11 are fundamentally flawed by their implicit assumption of strict geometric similarity disguised by constant measurement error.

In the earlier examination [10] of reported ultra-micro-indentation data [25] it was shown that recognition of an offset of zero in the P versus $F^{1/2}$ relationship (cf. Equation 3) resulted in diminished size effect and lower estimates of macrohardness. Re-examination of those data in the light of the present analysis reveals that the new description gives better accounts of the size effect. For aluminium, analysis according to Equation 8 gives the linear relationship shown in Fig. 13; whereas the P versus $F^{1/2}$ relationship was earlier shown to be distinctly non-linear. Calculation of macrohardness according to Equation 7 then yields more consistent and appropriately lower values. This is shown in Fig. 14, where the new result ("PH2") is compared with the earlier findings. In the case of quartz, the linear relationship according to Equation 8 is not obviously better than that from Equation 3, but estimated values of macrohardness are appreciably lower, as noted for aluminium (see Fig. 15). Thus, despite the obvious errors in values extracted from small graphs, it is clear that these data conform better to the present analysis than to the earlier simple recalibration.

Experimental error in the Vicker's test data appears to have been remarkably slight. Some of the residual variance, which appears to be random, no doubt arose from slight variation of the indented material between test sites. In this context the more precise relationships extracted from ultra-micro-indentation data for progressive indentation at one site are valuable confirmation of the accuracy of the new description of the size effect.

Similarity of the calculated values of δ for iron and for silica support the supposition that the calibration offset reflects conduct of the indentation tests rather than behaviour of the materials. It may be inferred,

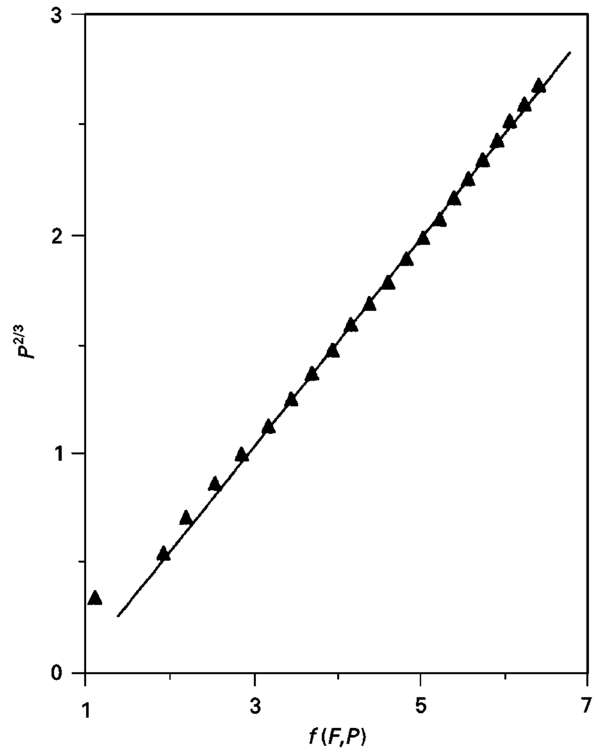

Figure 13 Calibration according to Equation 8. Data for aluminium from Ref. 25.

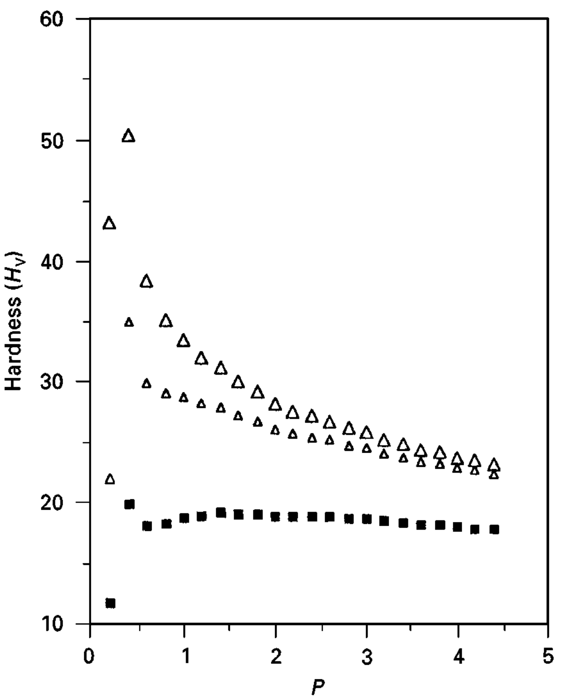

Figure 14 Hardness versus penetration for aluminium. Data from Refs 10 and 25. (△) original; (▵) corrected; (■) PH2.

from the fact that a calibration offset was only identified for ultra-micro-indentation tests, that this parameter signifies experimental errors related largely to initial contact. Apparently the small calibration offset effectively accounted for the several possible sources of error, such as imperfection of the extreme tip of the indenter and initial variation of the compliance of the test system, to which very fine depth measurements are prone. In principle the optically assessed Vicker's tests are not affected by system compliance nor unavoidably subject to indenter shape problems. (The mean indentation pressure varies only slightly between very blunt indenters [26, 27]). However, it is also to be expected that experimental error is less significant at this coarser scale.

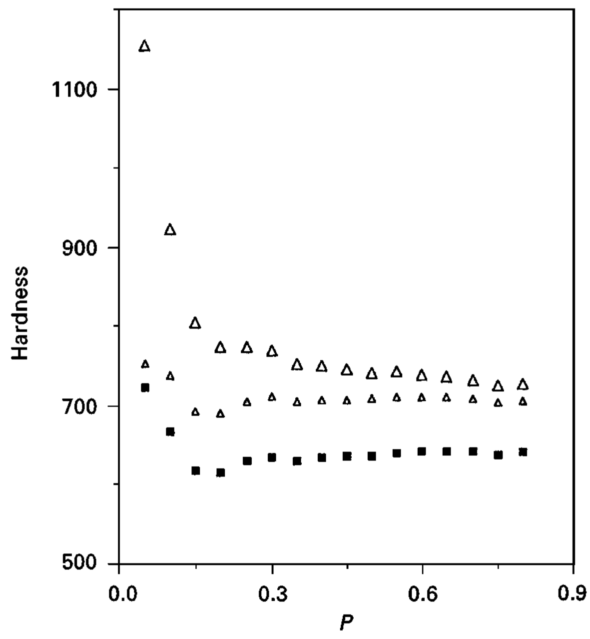

Figure 15 Hardness versus penetration for quartz. Data from Refs 10 and 25. (△) original; (▵) corrected; (■) H_0 (PH2).

Apart from the slight uncertainty at initial contact of the indenter in ultra-micro-indentation, the size effect was found to be completely and precisely described by the size effect parameter λ. Marked variation of this parameter between materials, while δ appears to be a system constant, indicates that the deformation behaviour of the indented material, rather than measurement errors, determines the magnitude of the size effect.

3.2. Indenter shape

The Berkovich indenter is designed to emulate the Vicker's hardness test and minimize problems due to malformation of the indenter tip. It's use is, therefore, particularly apposite in very fine depth-sensing indentation tests. Neglecting inaccuracies and elastic distortion of the indenters, the ratio of pyramid height to base area is the same for these indenters. If the emulation is satisfactory, it is meaningful to enquire whether the indentation size effect is continuous throughout the entire range of test conditions.

Unfortunately test data embracing this range for identical material were not available, but comparison of the ultra-micro-indentation of a very-low carbon steel with the indentation of pure iron in the higher load range reveals only a slight difference which is quite consistent with expected weak solution hardening. The estimated values of "characteristic macrohardness" for these materials indented under such disparate conditions were 87 H_V and 78 H_V respectively. (cf. Figs 7 and 10). The difference is just about what might be expected. The clear discrimination of this small difference is particularly significant: there is no overlap of the calculated values of macrohardness. Evidently the data correspond precisely in each case to a well-defined similar relationship.

Evaluations of the size effect parameters β and λ further this comparison. For the Vicker's tests of iron the calculated value of β was 1.9 $\mu m^{2/3}$, whereas for the ultra-micro Berkovich tests the value of λ was 55 $nm^{2/3}$. The two may be compared through the relationship (depending on units of measurement)

$$\frac{\lambda}{\beta} = \left(\frac{1000}{7}\right)^{2/3} = 27.37 \qquad (12)$$

from which it is seen that 1.9 $\mu m^{2/3} \cong 52\ nm^{2/3}$. Thus the apparent continuity of the size effect throughout the full range of indentation test conditions (See Fig. 3) is mirrored faithfully by closely corresponding values of β and λ. The evidence confirms the supposed equivalence of the indentation processes with these different shaped indenters and points unequivocally to continuous monotonic variation of apparent hardness with indent size.

3.3. Indentation

The identification of an accurate behavioural relationship corresponding to precisely constant value of H_0 is a special result unlikely to arise adventitiously. Equation 9, or ideally Equation 7, accounts for all the non-random variation in apparent hardness: it is so accurate that it is hard to believe an improvement would be possible. Therefore it is not unreasonable to suppose that it is descriptive of a causal relationship.

The relationship described by Equation 9 confirms that the indentation size effect is due to a cause that acts with monotonically diminishing effect over the large range of indent size employed in hardness testing. However, the form of this equation is not consistent with any known type of experimental error. Therefore the size effect is very probably a genuine variation of mean indentation pressure with scale of indentation. The implication is that either geometrical similarity of the indentation process is not maintained or the mean pressure is not determined by the strain pattern. The latter would signify variation of intrinsic strength according to scale of deformation, for a very large range of indentation, whereas loss of geometric similarity connotes merely a scale effect in the external constraint of deformation and friction is known to govern the size effect.

Indentation is a loading process in which deformation spreads to minimize stress (and to minimize local strain rate for rate-sensitive materials). The constrained flexing identified at the perimeter of an indent [16, 28, 29] interacts with the broad geometrically similar deformation induced by pressure from the indenter across an interface of constant geometry. This interaction is complex; and it is notable that, although the indentation size effect has been recognized for some ninety years, mathematical analyses of the deformation have not accounted for it [30–33]. No scale factor has been included.

Flexing at the perimeter of an indent presumably merges gradually into the surrounding broad pattern of deformation and is defined not by a definite boundary but, rather, by perturbation of the 'ideal' stress and strain gradients. As indentation proceeds, the strain field expands with approximately the same pattern of strain distribution; but the flexure zone evidently does not grow at a rate high enough to maintain strict geometric similarity of the overall deformation pattern. The edge of a large indent is, relatively, more sharply defined.

Equation 9 (or 7 for coarser scale of indentation) matches the predicted necessary refinement of the "plastic hinge" model of indentation [16], accommodating variation of the magnitude of this notional perimeter zone with indent size. Thus, the argument that the indentation size effect for metals is caused by the special boundary deformation conditions governed by strain hardening propensity and friction is much strengthened.

It is not obvious why the new size effect parameter, β or λ, has a dimension of $(\text{length})^{2/3}$. The perimeter "hinge" zone is quite complex, tapering from very little at each corner of the indent to maximum width at the mid sides where there has been most deformation. Moreover, the association of the size effect with strain hardening implies a correlation also with the shape of the indent boundary as modified by 'pile-up' or 'sink-in'. Comprehensive mathematical modelling would have to address these complications.

3.4. Friction

Li *et al.* [34] have interpreted reduction of the indentation size effect by lubrication in terms of their "proportional specimen resistance" model [35]. In this, the size effect is related to the varying ratio of contact area to 'displaced indentation volume' with indent size. The variation of apparent hardness is attributed to increased contribution of interfacial friction as the indent size becomes smaller. The essential differences between this model and the "plastic hinge", or perimeter flexing, model [16] lie in the assumptions concerning the indentation process and the role of friction. The former is based on constant specific work of deformation (implying geometric similarity of the deformation pattern) and friction affecting the whole contact area. The latter abandons geometric similarity and envisages a friction effect only at the perimeter. Thus the probability of strict geometric similarity is linked with the role of friction.

Several difficulties with the proportional resistance postulate undermine its choice as preferred model. First, the form of equation that divides the indenting force into components related to displaced volume or to contact area has no special status, since it has been shown to be precisely equivalent to Equation 3 – and inaccurate [8, 9]. The newly identified accurate description of the size effect is different dimensionally.

Second, the magnitude of the indentation size effect is strongly associated with the strain-hardening propensity of the indented material; and friction appears to be secondary to this association. Then again, the insensitivity, on the larger scale, of blunt indentations to lubrication is thought to indicate absence of general sliding across the interface. For friction to become important in small indentations, there must be significant sliding. This would be consistent with increased relevance of a normal edge effect.

Although a marked effect of lubrication, and by implication sliding friction, on the indentation size effect for soft metals has been demonstrated, Li *et al.* have recorded that an expected effect for non-metals has proved difficult to detect. Perhaps the higher mean pressure for indentation of the harder materials poses a lubrication problem, but the weaker size effect for hard non-metals indicates less friction, or less sliding. Association of the size effect with strain hardening is entirely consistent with an expectation of variable sliding at the perimeter of the indent. The unfailing correspondence of the perimeter "hinge" model with all the evidence sustains a very reasonable inference that the size effect is a boundary phenomenon and strict geometric similarity of the strain distribution is not maintained as indentation progresses. (Variation of friction according to the proportional resistance model would also seem to imply loss of geometric similarity.)

3.5. Materials

The indentation size effect for plastic metals and for brittle non-metals is similar in manner if not in magnitude; yet materials without multiple extensive slip systems, or free rheology, cannot accommodate indentation by a process corresponding to continuum plasticity. On the other hand, it would be a rather special material that could undergo indentation with a strain field corresponding exactly to that of a shear line field construction, including a point of singularity at the perimeter of the indent. Micrographs of very small indents commonly show marked departure from ideal shape.

Discrete striations of deformation damage have been observed on the surface of indents in ceramics which show an indentation size effect [27]. These markings were attributed to relief of stresses created by "elastic flexure of the surface at the edges of the indentation", and no doubt indicate a response to this constrained deformation equivalent to the "plastic hinge" postulated for metals. (Although some quasi-brittle materials are credited with plasticity, intermittent cracking creating discrete bands of deformation damage constrained by hydrostatic compression might be a more common mechanism that could sustain this relatively severe bending in a brittle material [36]). It may be supposed that, as another manifestation of the size effect, the spacing between surface striations would increase with indent size as the flexing strain gradients relax. This would furnish a nice demonstration of the interaction between constrained flexing at the perimeter and the wider strain field.

However, appearance is one thing, cause and effect another. Appeals to restriction of micro-deformation processes as an essential cause of the indentation size effect face a paradox: the most restricted and heterogeneous behaviour is associated with only small size effect. Soft metals exhibit both relatively uniform deformation (no deformation bands or cracking) and especially marked size effect. Evidently indentation, being a loading process, is fairly insensitive to the manner of distribution of enabling strains, but progressive resistance to strain is another matter. Strain hardening propensity is clearly the most important material property promoting the indentation size effect. Strain hardening opposes strain concentration, and therefore may be expected to have particular influence at the perimeter of an indent (where shear line fields show flexing to be concentrated) thus promoting a boundary effect.

Materials with severely restricted micro-plasticity may be expected to exhibit more complicated behaviour superimposed on the general effect. Discontinuities in the force-penetration relationships reported [29] for some brittle non-metals presumably represent this class of behaviour.

3.6 Conspectus

The finding that indentation size effects for both eminently plastic metals and brittle non-metals are amenable to the same analysis poses a need to examine concepts of "hardness". If hardness is defined as resistance to (a particular mode of) permanent deformation, then an ideally elastic material has infinite hardness. On the other hand, if hardness is defined as resistance to the process of indentation, then no distinction is made between elastic and permanent (by whatever mechanism) deformation. This is the hardness defined by the penetration-force relationship during indenting.

In this study, the calculation of H_0 from depth-sensing test data is the lower bound evaluation ignoring elastic recovery. Therefore H_0 is "low" in comparison with other evaluations when elastic deformation is significant. However, H_0 is probably not perceptibly low for soft plastic materials with high modulus of elasticity: i.e. metals. In fact this definition of hardness is entirely consistent with the principle of Vicker's testing, for which the measured dimensions across the "corners" of the indent are reckoned to be substantially free from elastic recovery, at least for metals. This agreement is confirmed by the correspondence between the data sets for iron tested with either optical measurement or depth sensing.

It has commonly been assumed that non-linearity of the P versus $F^{1/2}$ relationship (Equation 3) is caused by experimental inexactitude. Imperfection of the tip of the indenter is a particular concern for ultra-micro-indentation testing. However, as noted above, many other origins have been postulated. The problem has been one of distinguishing between the strengths of the various claims. And the interactions between the several factors – various modes of heterogeneous deformation, evident friction and possible error – make the problem quite complex. Nevertheless, analyses have often examined only data limited in quantity or precision, so that the outcome has inevitably been restricted to simplistic relationships.

The newly discovered precise relationship, based on adequate data embracing wide ranges of indent size and of material plasticity, establishes unequivocally a more complex form of the indentation size effect than hitherto envisaged. The wide range of the size effect sometimes encountered, coupled with relatively uniform deformation, suggests variation of the indentation process that probably stems from the fundamental mechanics rather than essentially from restriction of micro-deformation processes.

Indentation by a quasi-rigid blunt pyramidal indenter imposes similar constraints on the deformation of any material; and close conformity to the shape of the indenter at the line of entry necessitates bending and stretching. Except in the special case of strict geometric similarity, a discernible volume of this constrained flexing at the edge of the general strain field must produce, regardless of the actual mechanism, an indentation size effect. The unifying principle appears to be simply that boundary effects increase the specific resistance to small indentation. The limiting, asymptotic, macro-hardness is the indentation resistance when boundary effects are negligible.

Reduction of the indentation size effect by lubrication implies sliding friction, and this must occur at the perimeter but need not occur over the entire contact area. Variation of true contact area, due to "pile-up" or "sink-in", might also be important. However, association of magnitude of the size effect with the strength of strain hardening indicates that constrained flexing is more important than the secondary topological effects – because strain hardening enhances the size effect but promotes "sink-in", which produces apparent softening: not hardening. Gane and Cox [37] have shown pronounced edge rounding of very small indents in gold, and the atomic field microscope is revealing comparable effects for other materials.

The size effect is not dependent on any particular deformation mechanism, but the value of the parameter λ reflects the component of work attributable to this boundary flexing and thus indicates resistance to strain concentration (and the influence of friction in promoting the effect). Therefore the size effect parameter λ could be a useful indicator of strain hardening propensity for metals, and perhaps of fracture energy for brittle materials.

4. Conclusions

Variation of apparent hardness with size of indent (the "indentation size effect") in either soft iron or relatively brittle fused silica is matched very precisely by a new descriptive equation. The size effect parameter contained in this equation has dimension of $(\text{length})^{2/3}$. This finding is based on analysis of ample precise data to make it reliable.

Close correspondence between the relationships revealed by the new analysis for ultra-micro-indentation or for Vicker's indentation of iron demonstrates that the indentation size effect is monotonically continuous throughout the full range of test indentations.

The accuracy of the new account of the size effect and its applicability to different test methods rule out experimental error as an essential cause. Applicability to such different materials establishes a common phenomenology regardless of the different micro-mechanisms sustaining the deformation.

The large difference between the values of the new size effect parameter for iron and silica is in keeping with the previously recognized association between magnitude of the size effect and uniform strain hardening plasticity.

The revealed form of the size dependence casts doubt on the concept of strict "geometric similarity" as a fundamental characteristic of hardness tests with a blunt pyramidal indenter. Correlation of the magnitude of the size effect with both strain hardening propensity and lubrication points rather to a boundary effect varying in importance with scale.

The new description of the indentation size effect was derived empirically but is consistent with a projected refinement of the "plastic hinge" model of indentation proposed earlier. According to this model, the constrained flexing at the line of entry of the indenter particularly affects a narrow perimeter zone. It was predicted, and is now confirmed, that the size of this notional zone varies with indent size, but not in simple proportion.

A reasonable and self-consistent interpretation of the findings is that higher apparent hardness from finer indentation tests is caused by interference from a boundary effect governed by strain hardening propensity and, secondarily, by friction. This apparent sensitivity to strain concentration at the perimeter of the indent is reflected in the new size effect parameter λ, which could therefore offer a useful assessment of strain hardening propensity.

Although the magnitude of the size effect is, seemingly, governed by the deformation characteristics of the indented material, for the data examined there appears to be no change in behaviour at very fine scale. Clearly structurally, or micro-structurally, determined heterogeneity of deformation is not the prime cause of the size effect. However, materials with severely limited deformation capability may be expected to exhibit complex behaviour superimposed on the general size effect.

Appendix: the "plastic hinge" model of indentation [16].

Indentation of an elastic–plastic material is accommodated by complex deformation involving displacements throughout a relatively large volume. In principle, the pattern of this deformation is independent of scale. However, at the boundary of an indent the indented material must shear to conform to the new surface. See Fig. 16. The necessary local shear strain is determined by the geometry of indentation.

For an ideally plastic material (a non-work-hardening continuum), the boundary shear condition is concentrated in a vanishingly small volume (the point of

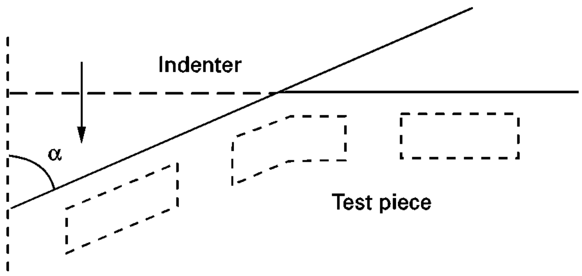

Figure 16 Shearing of surface element as indenter penetrates surface (from Ref. 16).

singularity in a shear line field construction) and indentations are 'geometrically similar': i.e. not subject to a scale effect.

For practical indentations in a real strain hardening material, strain concentration is resisted and the expanded boundary zone is significant, particularly when the indent is small. This relative significance is postulated to be the origin of the indentation size effect [16].

The plastic hinge model has been shown to offer a reasonable explanation for the dependence of the indentation size effect on strain hardening propensity and on friction.

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